Scienta Press Journals

American AgriTech
JOURNAL OF AGRICULTURAL TECHNOLOGY INNOVATION AND SUSTAINABILITY

NUMERICAL ANALYSIS OF WAVE PROPAGATION IN THREE-LAYER VISCOELASTIC RODS AND PLATES WITH A FILLER

Navruzbek Qalandarov, Muhammad Narzulloyev
Asia International University (Uzbekistan, Bukhara)

Abstract

This article presents numerical results for wave propagation problems in three-layer viscoelastic rods (beams) and plates with a filler. A dispersion equation is obtained for the flexural vibrations of a three-layer beam, and the variation of phase and group velocities with wavelength is analyzed for an aluminum-plastic pair; dispersion relations are also derived for a three-layer plate in a half-space and for the flexural vibrations of a two-layer coated elastic plate, with numerical results discussed for a duralumin-acrylonitrile pair. The asymptotics of the obtained numerical solutions near zero frequency are compared with known results for an elastic layer, and their agreement is presented as evidence of the reliability of the results obtained.

Keywords: Wave propagation, three-layer rod, dispersion equation, phase velocity, group velocity, damping coefficient, duralumin, acrylonitrile

Annotatsiya.

Ushbu maqolada to'ldiruvchili uch qatlamli qovushqoq-elastik sterjen (balka) va plastinkalarda to'lqin tarqalishi masalalarining sonli natijalari keltirilgan. Uch qatlamli balkaning egilish tebranishlari uchun dispersion tenglama olindi va Alyuminiy-plastik juftlik uchun faza va guruh tezliklarining to'lqin uzunligiga bog'liq o'zgarishi tahlil qilindi; yarim tekislikdagi uch qatlamli plastinka va ikki qoplamali elastik plastinkaning egilish tebranishlari uchun ham dispersion munosabatlar olinib, dyuralyumin–akrilon juftligi misolida sonli natijalar muhokama qilindi. Olingan sonli yechimlarning nolinchi chastota atrofidagi asimptotikalari elastik qatlam uchun ma'lum natijalar bilan solishtirilib, ularning mos tushishi natijalarning ishonchliligini tasdiqlovchi dalil sifatida keltirilgan.

Kalit so'zlar: to'lqin tarqalishi, uch qatlamli sterjen, dispersion tenglama, faza tezligi, guruh tezligi, so'nish koeffitsiyenti, dyuralyumin, akrilon.

Аннотация.
В данной статье приведены численные результаты решения задач распространения волн в трёхслойных вязкоупругих стержнях (балках) и пластинах с заполнителем. Получено дисперсионное уравнение для изгибных колебаний трёхслойной балки и проанализировано изменение фазовой и групповой скоростей в зависимости от длины волны для пары алюминий–пластик; также получены дисперсионные соотношения для трёхслойной пластины в полупространстве и изгибных колебаний двухслойной облицованной упругой пластины, численные результаты обсуждаются на примере пары дюралюминий–акрилон. Асимптотики полученных численных решений вблизи нулевой частоты сопоставлены с известными результатами для упругого слоя, их совпадение приводится как свидетельство достоверности полученных результатов.

Ключевые слова: распространение волн, трёхслойный стержень, дисперсионное уравнение, фазовая скорость, групповая скорость, коэффициент затухания, дюралюминий, акрилон.

INTRODUCTION

The preceding articles presented the mathematical formulation of the problem of wave propagation in a three-layer viscoelastic plate with a filler, together with the solution methodology, including the method for deriving the dispersion equation. The purpose of this article is to apply the developed methodology to three specific problems and to discuss the resulting numerical findings: (1) the flexural vibrations and wave propagation of a three-layer rod (beam); (2) wave propagation in a three-layer plate lying on a half-space; (3) the flexural vibrations of a two-layer coated elastic plate. For all three problems, dispersion equations were obtained and their roots solved numerically using Muller's and Gauss's methods, with the results analyzed for real structural materials (aluminum, plastic, duralumin, acrylonitrile).

1. Flexural vibrations of a three-layer rod (beam)

A three-layer rod (beam), infinite on both sides, made of a viscoelastic material, whose bending occurs in the xy-plane, is considered. The complex-coefficient wave equation for the displacement potentials (φ, ψ) is written as

$$G0\cdot[2/(1-v)]\cdot(1-i\Gamma\lambda\mu)\cdot\nabla^2\varphi = \rho\cdot\partial^2\varphi/\partial t^2, \quad G0\cdot(1-i\Gamma\mu)\cdot\nabla^2\psi = \rho\cdot\partial^2\psi/\partial t^2 \qquad (1)$$

where G0 is the instantaneous shear modulus and ρ is the density. Seeking the solution in the form $\varphi=\Phi(x,y)e^{i\omega t}$, $\psi=\Psi(x,y)e^{i\omega t}$ (2) yields the equations $\nabla^2\Phi+h^2\Phi=0$, $\nabla^2\Psi+k^2\Psi=0$ (3), where $h^2=(1-v)\rho\omega^2/[2G0(1-i\Gamma\lambda\mu)]$, $k^2=\rho\omega^2/[G0(1-i\Gamma\mu)]$. For a structure symmetric with respect to the middle plate, with two identical outer layers, the solution is sought in the form

$$\Phi1 = B1\cdot sh(\alpha1 y)\cdot\sin(\xi x), \quad \Psi1 = D1\cdot ch(\beta1 y)\cdot\cos(\xi x), \quad \Phi2 = [A2\cdot ch(\alpha2(c-y))+B2\cdot sh(\alpha2(c-y))]\cdot\sin(\xi x) \qquad (4)$$

($\alpha j^2=\xi^2-hj^2$, $\beta j^2=\xi^2-kj^2$). Imposing a stress-free condition on the outer surfaces (at y=±H, σy,2=σxy,2=0) and a continuity condition at the layer-filler contact (at y=±H1, σy,1=σy,2, σxy,1=σxy,2, u1=u2, ϑ1=ϑ2) produces a homogeneous system of linear algebraic equations for the unknown integration constants (B1,D1,A2,B2). The condition for this system to have a nontrivial solution — that its main determinant vanish — yields the dispersion (frequency) relation.

Introducing the phase velocity V=ω/ξ, the instantaneous longitudinal and transverse wave velocities are defined by

$$Vs0,j^2 = G0j/\rho j, \quad Vl0,j^2 = 2G0j/[\rho j(1-vj)], \quad j = 1,2 \qquad (5)$$

Depending on the ratio of these velocities, four distinct cases are possible (Vl0,2>Vs0,2>Vl0,1>Vs0,1, and so on), and for each of these the parameters αj and βj take real, complex, or imaginary values. After the corresponding substitutions, the resulting transcendental dispersion equation was solved numerically using Muller's and Gauss's methods.

Numerical computations were carried out for a three-layer beam with h2/h1=1/10, where the outer layer is aluminum ($GA=26.5\cdot10^9$ Pa, vA=0.33) and the middle (filler) layer is plastic ($Gp=1.1\cdot10^9$ Pa, vp=0.34). The results show that as the wavelength decreases the phase velocity tends to an asymptotic value; the two modes of phase velocity (real and imaginary parts) exhibit qualitatively different curves. On the ordinate axis, ReV=Re(V/Vs0,1) and −ImV=−Im(V/Vs0,1) are used, with modes 1,2 representing the real part of the group velocity and modes 3,4 the corresponding imaginary parts.

Analyzing the dependence of the group velocity on the wavenumber revealed that, at small wavenumbers (small values of $ReVg=Re(Vg/Vs0,1)$), it is a non-monotonic function. This finding indicates that the rate of energy transfer (the group velocity) in a three-layer beam does not follow a single monotonic law over the entire wavelength range — a fact that requires particular care when transmitting short-wave signals in practice. Thus, a dispersion equation was obtained and solved numerically for the problem of wave propagation in a three-layer beam, and the laws governing the variation of the group and phase velocities with wavelength were established.

2. Wave propagation in a three-layer plate on a half-space

A sequence of layers bounded by parallel planes in a Cartesian coordinate system is considered, filled with isotropic elastic media; rigid bonding or slip contact conditions are imposed at the layer boundaries, while the Sommerfeld radiation condition is satisfied at infinity. The dispersion equation obtained on the basis of the displacement potentials is written in general form as

$$|\beta ij| = 0, \quad i,j = 1,2,\ldots,12 \qquad (6)$$

a determinant of order 12, whose elements are composed of exponential functions such as $\beta_{1,1}=(1+\bar{s}_1^2)\cdot\exp(-kq_1h_1^*)$ $(h_1^*=h_1+h_2)$. This equation is solved numerically for ω/k at various values of k.

The computations were performed using the following dimensionless parameter values: $S^2L1=0.622$; $S^2L2=S^2L3=3.360$; $S^2S1=0.776$; $S^2S2=1.230$; $S^2S3=3.000$; $\mu_1=0.170$; $\mu_2=\mu_3=0.30$. The results were analyzed graphically for the dependence of the wavenumber ($\xi=kH$) on the phase velocity in a two-layer filled structure, and for the dependence of the natural frequency on the wavenumber (separately for dissipatively homogeneous and inhomogeneous systems). For a dissipatively homogeneous system, the damping coefficient δ is determined by the imaginary part of the first complex natural frequency, and the ω-ξ relationship is non-monotonic in character. For a dissipatively inhomogeneous system, the role of the global damping coefficient is played jointly by the imaginary parts of the first and second frequencies — the mathematical and physical aspects of this effect are explained in the existing literature. This result shows that, by purposefully changing the geometric dimensions or physical properties of the system, the damping properties of a dissipatively inhomogeneous viscoelastic system can be controlled.

3. Flexural vibrations of a two-layer coated elastic plate

A plate whose main layer occupies the region $-b/2 \le y \le b/2$ and whose identically-thick coatings occupy the regions $(-h+b/2 \le y \le -h;\ h \le y \le b/2+h)$ is considered, with its neutral plane coinciding with the xz-plane; harmonic flexural waves propagate along the z-axis. The solution for the main layer, obtained by the method of separation of variables for the Rayleigh–Lamb problem, is of the form $u=0$, $\vartheta=(\alpha \cdot A1 \cdot ch(\alpha y)-k \cdot C1 \cdot sh(\beta y)) \cdot e^{(i(\omega t-kz))}$, $w=(-ik \cdot A1 \cdot ch(\alpha y)+i\beta \cdot C1 \cdot sh(\beta y)) \cdot e^{(i(\omega t-kz))}$ (7), where

$$\alpha^2 = k^2 - \omega^2\rho/(\lambda G + 4\mu/3), \quad \beta^2 = k^2 - \omega^2\rho/\mu \tag{8}$$

A similar solution, with the appropriate indices, holds for the coating layer. From the stress-free condition at the free surface ($\sigma xy=\sigma yy=\sigma yz=0$) and the continuity of displacements and equality of stresses at the layer contact, a homogeneous system of linear algebraic equations is constructed with respect to six integration constants (A1, B1, C1, D1, F1, M1). The condition for this system to have a nontrivial solution, $F(\Omega)=|C|=0$ (9), yields the dispersion equation, a transcendental equation solved numerically by Muller's method (verified, where necessary, using Newton's method or a small-parameter method).

For the numerical computations, the following values were adopted for the middle layer (duralumin) and the coating (acrylonitrile): Poisson's ratio 0.30 and 0.25, respectively; longitudinal and transverse wave velocities Cpc=5400 m/s, Csc=3195 m/s (for the coating) and CLpk=2300 m/s, Cspk=1311 m/s (for the middle layer); the coating-to-middle-layer density ratio $\rho sr=\rho pk/\rho s=0.4452$; and the Rzhanitsyn–Koltunov kernel parameters describing the coating's rheological properties A=0.048, $\beta=0.05$, $\alpha=0.1$. The absolute error of the eigenvalues relative to the exact equations was approximately $14 \cdot 10^{-6}$ in the computations.

The results obtained showed the following: the imaginary part of the frequency varies with the thickness ratio (since the coating is elastic and the filler is viscoelastic, the structure is not dissipatively homogeneous); the damping coefficient depends non-monotonically on the wavelength, with the role of the global damping coefficient played by the imaginary parts of the first and second natural frequencies, which intersect as they approach each other (a similar effect was confirmed when using

Rabotnov's fractional-exponential kernel); the dispersion curves are symmetric with respect to the substitution $\tilde{k} \to -\tilde{k}$; and, for the hereditary-elastic spectrum, the concepts of "cutoff frequency" and "minimum frequency" characteristic of the elastic spectrum lose their meaning, since the corresponding dispersion equation has no roots at such points.

The asymptotic solution obtained for the first mode near zero frequency, $k1=c12\omega+c13\omega^{(3/2)}+O(\omega^2)$, $\alpha1=d13\omega^{(3/2)}+O(\omega^2)$ (10) (where c12, c13, d13 are functions of ν, m, β), was shown to coincide completely, for m=0, with the known asymptotics for an elastic layer. The agreement between the numerical and asymptotic solutions is an important piece of evidence confirming the reliability of the results obtained.

Graphical analysis of the dispersion curves also included the projections, on the (ω*, k) plane, of k* for both positive and negative imaginary parts, and projections on the (ω*, α) plane for various values of the parameters m and β expressing the material's hereditary properties. Numerical analysis showed that the larger the value of m, or the smaller the value of β, the earlier and more rapidly the dispersion curves for k* with positive and negative imaginary parts diverge from one another; conversely, as m decreases or β increases, the dispersion curves tend toward the elastic case, i.e., the elastic spectrum can be regarded as the asymptotic limit of the hereditary-elastic spectrum as $k \to 0$, $\beta \gg 1$. This conclusion is of practical importance: the higher the degree of viscosity of the filler (expressed through the parameter β), the further the dynamic properties of the structure deviate from the elastic approximation, meaning that the error introduced by simplifying a viscoelastic calculation to an elastic one also increases accordingly.

CONCLUSION

This article presented dispersion equations, obtained and numerically analyzed, for three specific forms of the wave propagation problem in three-layer viscoelastic rods and plates with a filler (a flexural beam, a plate on a half-space, and a coated plate). In all three problems, the symmetry of the dispersion curves under the substitution $\tilde{k} \to -\tilde{k}$, together with the fundamental differences between the hereditary-elastic and elastic spectra (the loss of meaning of the cutoff frequency and minimum frequency concepts), was established as a general pattern.

The complete agreement of the asymptotic solutions near zero frequency with the known results for the corresponding elastic problem, together with the small computational error (of order ~$14 \cdot 10^{-6}$) of the eigenvalues relative to the exact equations, confirms the reliability of the developed numerical methodology and the results obtained. These results form the basis for the following article, which analyzes the dynamic stress-strain state under harmonic loading and the associated practical recommendations, including the reduction of the resonance amplitude.

REFERENCES

1. Болтаев З.И. Распространение линейных гармонических волн в протяженных плоских и цилиндрических телах с учетом вязкоупругих свойств материала. Ташкент: Фан, 2013. -136с.
2. Болтаев З.И. Передача подвижной нормальной нагрузки на упругую среду через амортизирующий слой // Узбекский журнал Проблемы механики. Ташкент, 2011. N3-4. С.9-12.
3. Гринченко В.Т, Комиссарова Г.Л. Свойства локализованных вблизи границ волновых движений в заполненном жидкостью цилиндре // Акустичний вiсник. 2006. Том 9, N 2. С. 37-55.
4. Султанов К.С. Распространение продольных волн в вязкоупругом полупространстве, включающем поглощающий слой // ПМТФ, №5, 1984. С.137-142.
5. Safarov I.I, Akhmedov M.Sh, Boltaev Z.I. Distribution of the natural waves of extended Lamellar viscoelastic bodies of variable thickness. Германия, LAP Lambert Academic Publishing, 2015. -120 p.
6. Safarov I.I, Boltaev Z.I, Axmedov M.Sh. Waveguide Propagation in Extended Plates of Variable Thickness // Open Access Library Journal, 2014. P.2-9.
7. Gibson R.F. Dynamic Mechanical Properties of Advanced Composite Materials and Structures: A Review // Shock & Vibration Digest. 1987. Vol. 19, No. 7. P. 13-22.