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# Dynamic Stress-Strain State Of Three-Layer Viscoelastic Plates Under Harmonic Loading And Reduction Of The Resonance Amplitude

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**Abstract**

This article investigates the dynamic stress-strain state (DSSS) of a three-layer plate lying on a viscoelastic n-layer deformable half-space under a vertical harmonic load. The concept of the global resonance amplitude (GRA) is introduced, a numerical analysis of the frequency dependence of the stress amplitude (frequency response) is performed for dissipative-homogeneous and inhomogeneous mechanical systems, and the influence of the middle layer stiffness (Efill/Epl) on the resonance amplitude is established. For an acrylonitrile-duralumin pair, the maximum stress amplitude is shown to decrease from 3.2 MPa to 0.94 MPa at the "role-switching" point. A practical implementation result of the developed methodology is presented — a reduction of up to 27% in the resonance amplitude of a 24 NDS (D6300-80) pump unit.

**Keywords:** dynamic stress-strain state, global resonance amplitude, frequency response, three-layer plate, resonance amplitude reduction.



**Annotatsiya.**

Ushbu maqolada qovushqoq-elastik n qatlamli deformatsiyalanuvchi yarim tekislikda yotuvchi uch qatlamli plastinkaning vertikal garmonik yuk ta'siridagi dinamik kuchlanganlik-deformatsiyalanganlik holati (DK-DH) o'rganilgan. Global rezonans amplitudasi (GRA) tushunchasi kiritilgan, dissipativ bir jinsli va bir jinsli bo'lmagan mexanik sistemalar uchun kuchlanish amplitudasining chastotaga bog'liqligi (AChX) sonli tahlil qilingan, o'rta qatlam bikirligi (Eto'ldir/Epla) ning rezonans amplitudasiga ta'siri aniqlangan. Akrilon-dyuralyumin juftligi misolida maksimal kuchlanish amplitudasi 3,2 MPa dan "rollarni almashish" nuqtasida 0,94 MPa gacha kamayishi ko'rsatilgan. Ishlab chiqilgan uslubiyotning amaliy tatbiqi — 24 NDS (D6300-80) rusumli nasos agregatining rezonans amplitudasini 27% gacha kamaytirish natijasi — keltirilgan.

**Kalit so'zlar:** dinamik kuchlanganlik-deformatsiyalanganlik holati, global rezonans amplitudasi, amplituda-chastota xarakteristikasi, uch qatlamli plastinka, rezonans amplitudasini kamaytirish.

**Аннотация.**
В данной статье исследовано динамическое напряжённо-деформированное состояние (ДНДС) трёхслойной пластины, лежащей на вязкоупругом n-слойном деформируемом полупространстве, при действии вертикальной гармонической нагрузки. Введено понятие глобальной резонансной амплитуды (ГРА), выполнен численный анализ зависимости амплитуды напряжений от частоты (АЧХ) для диссипативно-однородных и неоднородных механических систем, установлено влияние жёсткости среднего слоя (Езап/Епл) на резонансную амплитуду. На примере пары акрилонитрил-дюралюминий показано снижение максимальной амплитуды напряжений с 3,2 МПа до 0,94 МПа в точке «смены ролей». Приведён практический результат внедрения разработанной методики — снижение резонансной амплитуды насосного агрегата марки 24 НДС (Д6300-80) до 27%.

**Ключевые слова:** динамическое напряжённо-деформированное состояние, глобальная резонансная амплитуда, амплитудно-частотная характеристика, трёхслойная пластина, снижение резонансной амплитуды.

## INTRODUCTION

In modern mechanical engineering and construction, many structural elements are in a dynamic stress-strain state (DSS) as a result of various dynamic loads, so that accurately computing the DSS requires accounting for the medium surrounding such structures. Much of shipbuilding, aircraft construction, and moving transport consists of layered polymer plates and plates containing fluid, in which the dissipation of the resulting vibrations obeys the laws of dissipatively inhomogeneous mechanical systems, according to the structure of the construction. The problems of studying the DSS arising from vibration of structures in contact with a medium, and of road and airfield pavement elements, likewise obey these same laws, making their reduction (optimization) an urgent scientific and practical task.

Building on the problem statement, solution methodology and dispersion equations developed in the preceding articles, this article presents a numerical analysis of the DSS of a viscoelastic three-layer plate under harmonic loading, demonstrates, on the basis of the global resonance amplitude concept, the possibilities for reducing the resonance values of displacements and stresses, and presents the results of the practical implementation of the developed methodology.

## 1. Problem statement

Let a dissipative mechanical system consisting of a viscoelastic n-layer deformable half-space be subjected to a vertical harmonic load r(x,t). Each layer satisfies the operator form of the Lame coefficients:

$$\tilde{\lambda}_k f(t) = [v_k E_{0k}/((1+v_k)(1-2v_k))]\cdot[f(t) - \int_{-\infty}^{t} R_{\lambda k}(t-\tau)f(\tau)d\tau], \quad \tilde{\mu}_k f(t) = [v_k E_{0k}/(2(1+v_k))]\cdot[f(t) - \int_{-\infty}^{t} R_{\mu k}(t-\tau)f(\tau)d\tau] \quad (1)$$

Splitting the displacement vector into potential and solenoidal parts (the Green–Lamb transformation), the wave potentials for layer k satisfy the following integro-differential equations:

$$\nabla^2\varphi_k - \int_{-\infty}^{t} R_k(t-\tau)\nabla^2\varphi_k(\tau)d\tau = (1/c_{pk}^2)\cdot\partial^2\varphi_k/\partial t^2, \quad \nabla^2\psi_k - \int_{-\infty}^{t} R_k(t-\tau)\nabla^2\psi_k(\tau)d\tau = (1/c_{sk}^2)\cdot\partial^2\psi_k/\partial t^2 \quad (2)$$

where $c_{pk}^2=(\lambda_k+2\mu_k)/\rho_k$, $c_{sk}^2=\mu_k/\rho_k$. When a rigid bonding condition (7) holds between the layers, the boundary conditions are written as

$$\sigma_{33}^{1}(x,0,t) = -p(x)e^{(-i\omega t)}, \quad \sigma_{13}^{1}(x,0,t) = 0, \quad \sigma_{33}^{k}(x,h_k,t) = \sigma_{33}^{k+1}(x,h_k,t), \quad \sigma_{13}^{k}(x,h_k,t) = \sigma_{13}^{k+1}(x,h_k,t), \quad u_k(x,h_k,t) = u_{k+1}(x,h_k,t) \quad (3)$$

(k=1,2,...,n). Seeking the solution in the form $\varphi_k=F_k(z)e^{(i(\gamma x-\omega t))}$, $\vec{\psi}_k=\vec{\Psi}_k(z)e^{(i(\gamma x-\omega t))}$ (4) yields second-order complex-coefficient differential equations $\nabla^2F_k+\alpha_k^2F_k=0$, $\nabla^2\Psi_{xk}+\varpi_k^2\Psi_{xk}=0$ (5), where $\alpha_k^2=\omega^2/[s_{rk}^2(1-G_k(\omega))]$, $\varpi_k^2=\omega^2/[s_{sk}^2(1-G_k(\omega))]$, with $G_k$ a complex function expressed through the Rzhanitsyn–Koltunov kernel. The resulting complex-coefficient system of ordinary differential equations for the displacement potentials, through the introduction of a function F, is reduced to a fourth-order equation:

$$d^4F_k/dz^4 + L_{5k}\cdot d^2F_k/dz^2 + L_{6k}\cdot F_k = 0 \quad (6)$$

where $L_{5k}$ and $L_{6k}$ are coefficients expressed through the layer parameters and frequency. The solution of (6) is of the form $F_k(y)=A_k\cdot e^{(\alpha_k y)}+B_k\cdot e^{(-\alpha_k y)}+C_k\cdot e^{(s_k y)}+D_k\cdot e^{(-s_k y)}$ (7), with the arbitrary constants determined from the boundary conditions (3). This yields a system of inhomogeneous complex-coefficient linear algebraic equations with respect to the unknown constants, $[C]\{q\}=\{p\}$ (8), where $\{p\}$ is a column matrix consisting of the applied external vertical force. This system is solved using the Gauss method.

## 2. Numerical results and their analysis

Numerical results were obtained for a three-layer plate in two cases: (1) a dissipatively homogeneous mechanical system (all layers viscoelastic, with the relaxation kernel amplitudes differing between them); (2) a dissipatively inhomogeneous system (two identical outer layers elastic, with a viscoelastic middle filler layer). The following values were adopted in the computations: $\rho_{layer}/\rho_{polymer}=1.333$; $\nu_1=0.35$; $\nu_2=0.25$; Rzhanitsyn–Koltunov kernel parameters $A=0.048$, $\beta=0.05$, $\gamma=0.1$, $\alpha=0.1$; the physical and mechanical properties of the outer (upper and lower) plates were taken to be identical. It was earlier shown by F.B. Badalov and his students that the influence of the remaining auxiliary parameters on the displacements and stresses does not exceed 5-10%.

The dependence on frequency of the stress amplitude $|A_{\sigma 33k}|=|\sigma_{33}(x,z)/\sigma_{33p}(x,0)|$, characterizing the DSS at the point $z=h=1$, $x=0$ of the mechanical system, was studied. Vibrations of a layer and a half-space under harmonic loading have previously been investigated by a number of researchers [1-4]. To fully reduce the resonance amplitude of the mechanical system, the concept of the global resonance amplitude (GRA) was introduced:

$$\delta_{\sigma 33k} = \max_{\gamma}(|A_{\sigma 33k}|) \tag{9}$$

In a dissipatively homogeneous system, the role of the GRA is played by the resonance amplitude corresponding to the first frequency [5]. In a dissipatively inhomogeneous system, however (since the outer layers are elastic), the role of the GRA is played jointly by the resonance amplitudes corresponding to the first and second frequencies; the "switching of roles" (SR) point occurs where the real parts of the frequencies are maximally close and the global damping coefficient reaches an extreme value — and it is precisely at this point that $\delta_{\sigma 33k}$ attains its minimum value.

Computations were carried out for ratios of the instantaneous stiffness modulus of the middle (filler) layer to that of the outer layer, Efill/Epl, of 0.95, 0.75 and 0.45. The results show that a decrease in the ratio Efill/Epl leads to a decrease in the resonance amplitude — that is, a softer filler layer attenuates the resonance phenomenon. At Efill/Epl=0.45, the global damping coefficient (GDC) reaches its maximum value, so the resonance stress amplitude of the outer plates is minimal at this point.

As an example, consider the case of an acrylonitrile layer lying on a duralumin half-space: srI=2300 m/s, ssI=1311 m/s, vI=0.35 (acrylonitrile layer); srII=5400 m/s, ssII=3195 m/s, vII=0.30 (duralumin half-space); h=1 m, r=1500 N. The numerical results obtained show that, for the dissipatively inhomogeneous case, the maximum stress amplitude decreases from σ33=3.2 MPa to σ33=0.94 MPa at the "switching of roles" (SR) point — a decrease of nearly 3.4 times. It was also found that, in the small-wavenumber region (γh<0.2), the values obtained may be 20-30% higher than in the large-wavenumber region (γh>1); this pattern also holds for the displacement components.

On the basis of the numerical results obtained, the following conclusions were formulated: a methodology for reducing, by several times, the resonance amplitude of the displacements and stresses arising under external harmonic forces was developed and justified; the concept of the global resonance amplitude was introduced, with its physical significance confirmed through numerical experiment; and a 20-30% difference in the frequency-response values between the small- and large-wavenumber regions was established.

## 3. Practical implementation

The developed methodology and method for reducing the resonance amplitude have been tested in practice. It was applied within the framework of the OT-F4-01 fundamental project "Developing methods for studying and advancing the theory of the nonlinear dynamic stress-strain state of curved sections of multilayer composite pipes conveying viscous fluid under thermal and dynamic loading," carried out under the State Scientific and Technical Program at the Tashkent Institute of Chemical Technology (reference letter of the Tashkent Institute of Chemical Technology No. 1/01-2410 dated 8 July 2025); as a result, it became possible to manage energy in multilayer composite pipes conveying viscous fluid under dynamic loading.

The method for reducing the resonance amplitudes of the displacements and stresses arising in a mechanical system under external forces was used by the "Amu-Bukhara Machine Canal Administration" to reduce the resonance displacement amplitude of mechanisms (reference letter No. 4214, 04/19, of the Ministry of Water Resources of the Republic of Uzbekistan, dated 30 July 2025). As a result, the resonance amplitude of a 24 NDS (D6300-80) pump unit was reduced by up to 27%. This result demonstrates that the developed theoretical methodology and numerical algorithm can be applied to real structures and equipment, and that the theoretical conclusions obtained have been confirmed in real technical systems.

## CONCLUSION

This article examined the dynamic stress-strain state of a viscoelastic three-layer plate under vertical harmonic loading. The concept of the global resonance amplitude was introduced, and its physical significance and method of computation were substantiated for both dissipatively homogeneous and inhomogeneous mechanical systems. The numerical analysis showed that, by purposefully selecting the stiffness of the middle (filler) layer, the resonance stress amplitude can be reduced by several times (approximately 3.4-fold in the example considered) — an important result for structural design practice.

Two real practical implementations of the developed methodology were presented, confirmed by official reference letters — for composite pipes within the fundamental project of the Tashkent Institute of Chemical Technology, and for a 24 NDS (D6300-80) pump unit at the "Amu-Bukhara Machine Canal Administration" (a resonance amplitude reduction of up to 27%). This confirms the consistency between the theoretical results presented across the four articles of this series, their numerical analysis, and their practical implementation, and thereby the overall reliability of the research results.

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